A THERMOELASTIC CONTACT PROBLEM ON STRIP BENDING WITH ALLOWANCE FOR THICKNESS DEFORMABILITY

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The bending of a strip on a heated half-space is considered on the assumption that the strip deforms in response to the temperature distribution and the force of a distributed load.

We consider the planar problem of strip bending (Fig. 1) on an elastic half-space in which the temperature distribution is defined by

$$\Phi(x,\ y,\ z)\equiv\Phi(x,\ z)=\frac{T_2}{\pi}\left(\operatorname{arctg}\frac{a(1-x)}{z}+\operatorname{arctg}\frac{a(1+x)}{z}\right), \tag{1}$$

and the upper plane of the strip is subject to an external load $q(x, y) = q(x)$ and a temperature $T(x, y) = T_1 = \text{const}$; to determine the buckling we find the displacement u_3. We use Hooke's law incorporating the temperature:

$$\sigma_{ij}=\lambda_1\theta\delta_{ij}+2\mu_1 e_{ij}-(3\lambda_1+2\mu_1)\,\alpha_1 T\delta_{ij}. \tag{2}$$

We substitute these expressions into the Cauchy equilibrium equations to get

$$(\lambda_1+\mu_1)\,\theta_{,i}+\mu_1\Delta u_i-\alpha_1(3\lambda_1+2\mu_1)\,T_{,i}=0. \tag{3}$$

The following is [1] a particular solution:

$$u_i=\psi_{,i}, \tag{4}$$

where ψ is the thermoelastic potential and is the solution to

$$\Delta\psi=\frac{3\lambda_1+2\mu_1}{\lambda_1+2\mu_1}\,\alpha_1 T. \tag{5}$$

We assume that the function T is linear in z and that the loads applied to the bounding planes are constant: $q(x) = q$ and $p(x) = p$. Then the following is the particular solution to (5):

$$\psi=\frac{3\lambda_1+2\mu_1}{\lambda_1+2\mu_1}\,\alpha_1\left(\frac{Az^2}{2}-\frac{Bz^3}{6}\right). \tag{6}$$

From (6) and (2) we have the following value for σ_{33}:

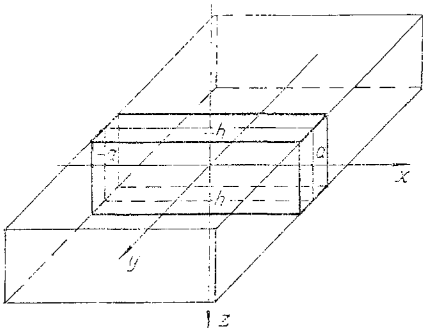
$$\sigma_{33}=(3\lambda_1+2\mu_1)\,\alpha_1(A+Bz)-(3\lambda_1+2\mu_1)\,\alpha_1 T(z). \tag{7}$$

We satisfy the boundary conditions at the upper and lower planes of the strip to get expressions for A and B:

$$\begin{aligned}A&=\frac{T_1+T_2}{2}-\frac{1}{2\alpha_1(3\lambda_1+2\mu_1)}\,(q+p),\\ B&=\frac{T_2-T_1}{2h}-\frac{1}{2h\alpha_1(3\lambda_1+2\mu_1)}\,(p-q).\end{aligned} \tag{8}$$

Lenin Belorussian University, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 6, pp. 1009-1012, June, 1983. Original article submitted April 13, 1982.

0022-0841/83/4406-0697$07.50



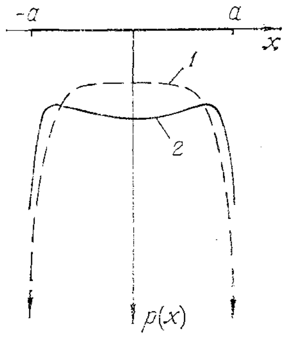


Fig. 1   Fig. 2

Fig. 1. Problem geometry.

Fig. 2. Distribution of the reactive pressure under a heated beam loaded by a uniformly distributed load: 1, 2) reactive pressure without and with allowance for the buckling, correspondingly: $E_1 = 2\cdot 10^3$ kg/mm^2; $E_2 = 2\cdot 10^2$ kg/mm^2; $q = 1$ kg/mm^2; $\varepsilon = 0.2$; $T_1 = 0$; $T_2 = 10°C$; $\alpha_1 = 1\cdot 10^{-5}$ 1/deg; $\alpha_2 = 5\cdot 10^{-6}$ 1/deg.

The approach between the bounding planes (buckling) is

$$w_b = u_3(-h) - u_3(h) = \frac{h}{\lambda_1 + 2\mu_1}(p+q) - \frac{3\lambda_1 + 2\mu_1}{\lambda_1 + 2\mu_1}\alpha_1 h (T_1 + T_2) = \\ = \frac{(1-2\nu_1)(1+\nu_1)h}{E_1(1-\nu_1)}(p+q) - \frac{1+\nu_1}{1-\nu_1}\alpha_1 h (T_1 + T_2). \tag{9}$$

We use the method of [2, 3] to show that (9) in the case of variables p, q, T has accuracy order $O(h^4/a^4)$.

The temperature in the strip varies linearly, so the thermal stresses are zero [4], and therefore the normal deflection of the points in the median plane of the plate w(x) satisfies

$$\frac{D}{a^4}\frac{d^4 w(x)}{dx^4} = q(x) - p(x) \tag{10}$$

and the boundary conditions

$$w''(x)|_{x=\pm 1} = w'''(x)|_{x=\pm 1} = 0. \tag{11}$$

From (10), (11) we get

$$w(x) = \frac{a^4}{D}\int_{-1}^{1}|x-t|^3[q(t) - p(t)]\,dt + C_1 + C_0 x. \tag{12}$$

On the other hand, the normal displacement of points in the median plane is equal to the sum of the displacements of the points at the boundary of the half-space [5] and half of the buckling, so according to [5] on the basis of (1), (9) we have

$$w(x) + \text{const} = \frac{\alpha_2 a(1+\nu_2)}{\pi}\int_{-1}^{1} T_2 \ln(a|x-t|)\,dt + \frac{2(1-\nu_2^2)a}{\pi E_2}\int_{-1}^{1}\ln(a|x-t|)\,p(t)\,dt + \frac{w_b}{2}. \tag{13}$$

We combine (12) and (13) to get the following integral equation for the contact pressure:

$$\varepsilon p(x) + \int_{-1}^{1}[\beta\ln(a|x-t|) + \gamma|x-t|^3]\,p(t)\,dt = f(x). \tag{14}$$

Here

$$\varepsilon=\frac{h}{a};\quad \beta=\frac{4(1-\nu_1)(1-\nu_2^2)E_1}{(1-2\nu_1)(1+\nu_1)\pi E_2};\quad \gamma=\frac{(1-\nu_1)^2a^3}{(1-2\nu_1)h^3};$$

$$f(x)=C_1+C_0x-\varepsilon q(x)-\varepsilon\frac{E_1\alpha_1}{1-2\nu_1}(T_1+T_2)+\gamma\int_{-1}^{1}|x-t|^3q(t)\,dt-$$

$$-\frac{2E_1\alpha_2(1+\nu_2)T_2}{(1-2\nu_1)(1+\nu_1)}\{(1+x)[\ln a(1+x)-1]+(1-x)[\ln a(1-x)-1]\}.$$

The arbitrary constants C_0 and C_1 are determined from the equilibrium conditions for the strip:

$$\int_{-1}^{1}p(x)\,dx=\int_{-1}^{1}q(x)\,dx, \tag{15}$$

$$\int_{-1}^{1}xp(x)\,dx=\int_{-1}^{1}xq(x)\,dx. \tag{16}$$

The solution to (14) is found in the form

$$p(x)=\sum_{k=1}^{N}d_k\varphi_k(x), \tag{17}$$

where

$$h\varphi_k(x)=\begin{cases}x-x_{k-1}, & x_{k-1}\leqslant x\leqslant x_k,\\ x_{k+1}-x, & x_k\leqslant x\leqslant x_{k+1},\\ 0, & x<x_{k-1}\ \text{and}\ x_{k+1}<x,\end{cases} \tag{18}$$

$$h=\frac{2}{N-1}.$$

From (14)-(16) we get by the collocation method a system of linear algebraic equations for the coefficients d_k ($k=\overline{1, N}$) and the arbitrary constants C_0 and C_1. An algorithm has been devised and a program has been written for the ES computer to solve this system of linear algebraic equations for any number of nodes. Numerical calculations imply that allowance for the buckling results in 20% (and more) error for the reactive pressure at the center, and also in finite values for this pressure at the edges (Fig. 2).

The above method can be transferred directly to other boundary-value problems for a strip.

NOTATION

Φ, temperature field; T_1, T_2, temperatures of the upper and lower planes; q, external load; u_i, displacement vector components; σ_{ij}, stress tensor components; e_{ij}, deformation tensor components; $\theta=e_{11}+e_{22}+e_{33}$; δ_{ij}, Kronecker symbol; λ, μ, Lamé coefficients; α, linear expansion coefficient; p, reactive pressure; D, cylindrical rigidity; Δ, Laplace operator; E, Young's modulus; ν, Poisson's ratio; $2a$, width; 2h, thickness. Elastic constants with subscript 1 refer to strip, those with 2 to half-space.

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